

*Citation for published version:*

Cross, B, Cripps, R, Hunt, M & Mullineux, G 2015, 'Singularities and 5-axis machining', Paper presented at ICMR2015 - 13th International Conference on Manufacturing Research, Bath, UK United Kingdom, 8/09/15 - 10/09/15 pp. 45-50.

*Publication date:*  
2015

*Document Version*  
Early version, also known as pre-print

[Link to publication](#)

**University of Bath**

**Alternative formats**

If you require this document in an alternative format, please contact:  
[openaccess@bath.ac.uk](mailto:openaccess@bath.ac.uk)

**General rights**

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

**Take down policy**

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

## **SINGULARITIES AND 5-AXIS MACHINING**

Ben Cross  
School of Mechanical Engineering  
University of Birmingham  
Birmingham  
B152TT, UK  
b.cross@bham.ac.uk

Robert Cripps  
School of Mechanical Engineering  
University of Birmingham  
Birmingham  
B152TT, UK  
r.cripps@bham.ac.uk

Mat Hunt  
Department of Mechanical Engineering  
University of Bath  
Bath  
BA2 7AY, UK  
m.hunt@bath.ac.uk

Glen Mullineux  
Department of Mechanical Engineering  
University of Bath  
Bath  
BA2 7AY, UK  
g.mullineux@bath.ac.uk

### **ABSTRACT**

Singular configurations of a tool path in five-axis CNC machine tools can cause undesirable machine behaviour. This paper examines what causes this behaviour by modelling the effect on machine axes movements. The model not only confirms the divergent characteristics at the singularity but provides a measure on the effect. The speed required to maintain a constant cutting feed rate is shown to depend upon the closeness to the singularity by a divergent factor. This provides insight into how a singularity avoidance strategy should be approached. A global reorientation strategy is then proposed for singularity avoidance. This has the effect of relocating the singular orientation into a prescribed safe direction. Justification for a singularity relocation is then argued via the effect on machine axes movement.

**Keywords:** five-axis machine tool, singularity, reorientation strategy

### **1 INTRODUCTION**

Singular configurations of a five-axis machine tool exist when there is a non-zero input that yields a zero output (Zlatanov 1994). That is to say the output, the position and orientation of the cutter with respect to the workpiece, is unaffected by certain inputs, machine axes movements. Configurations that are within some tolerance of a singularity then form a singular region. When the machine configuration enters this singular region machine axes movement becomes less predictable and unstable (Affouard *et al.* 2004).

This can be caused by an under-sampling of the CAM tool path data in the post-processing stage (Sørby 2007). Although an improvement in accuracy can be achieved by increasing the sampling local to the singularity, the speed of machine axes has to be increased to maintain a constant cutting feed rate. If the configurations are too close to the singularity then restrictions on the machine tool inhibit faster axes movement and consequently the cutting feed rate decreases. As a result of this behaviour undesirable cutter dwell marks and an increase in machining time are observed (Lin *et al.* 2014).

To overcome these concerns, singularity avoidance strategies have been developed. These involve a local manipulation of the tool path near to the singularity at the CAM stage (Lin *et al.* 2014, Affouard *et al.* 2004, Yang and Altintas 2013). However this approach raises concerns for the machining strategy. Firstly, the tool has to be reoriented to preserve the cutter contact point. Ball end mills may be reoriented with relative simplicity. Reorientations for flat end and small radius tipped mills on the other hand may not be possible or may cause gouges. These new configurations could also cause machine-workpiece collisions.

The fragmentary nature of choosing only to manipulate the tool paths local to the singularity also has a downstream effect on surface quality. Local reorientations change the angle between the cutter and the surface normal, the yaw angle, and the angle between the cutter and direction of motion, the tilt angle. This in turn affects the chip pattern on the surface of the workpiece (Lavernhe 2010). If adjustments are made locally around a singularity, there is a visible change in the texture of the finished surface (Lin *et al.* 2014). Such changes are undesirable and ideally should be avoided.

This paper proposes a global reorientation strategy for singularity avoidance that is not affected by the concerns raised previously for local manipulations. The solution is simple in that it involves only two additional steps. Firstly, a jig is required to reorient the workpiece on the machine bed. Secondly, the CAM tool path needs to be reoriented. This has the effect of relocating the singular configurations to safe positions.

It is unstable machine axes movements which characterise the undesired behaviour at the singularity. Therefore these movements are modelled by considering the differences between the coordinate systems that define the machine configuration and the position and orientation of the cutter relative to the workpiece. These models provide insight into what the effect of traversing through singular regions has on machine axes movement. This motivates a strategy for relocating the singularity to be as far as possible from the tool path.

In outline the paper is organised as follows. Section 2 introduces the coordinate systems and derives the model for predicting machine axes movements. Section 3 proposes a global reorientation strategy to avoid singularities with the use of a jig. In section 4 an example is presented to demonstrate the ideas. Finally section 5 finishes with some concluding remarks on singularities.

## 2 MACHINE AXES MOVEMENTS NEAR TO THE SINGULAR CONFIGURATION

To begin two coordinate systems (frames) are introduced to distinguish between the CAM tool path and the actual machining stage. *Workpiece coordinates* describe the position and orientation of the cutter with respect to the workpiece. *Machine coordinates* describe the configuration of the machine tool axes. The two frames are connected via a kinematic chain of transformations. The analysis presented here takes on a specific kinematic chain for simplicity, which is based on the Hermle C600U machine tool (Hermle 1999). The five axes consist of three translational axes ( $X$ ,  $Y$ ,  $Z$ ) controlling spindle position and two rotary axes ( $A$ ,  $C$ ) controlling the orientation of the workpiece. The workpiece is fixed onto a rotary table ( $C$ -axis) that is attached to a tilting table ( $A$ -axis) at the lower part of the machine (Figure 1).

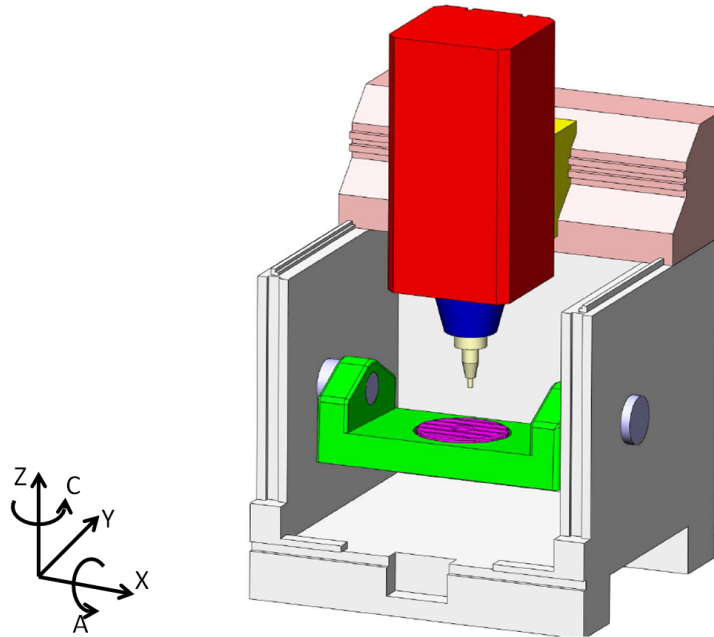


Figure1. Schematic of the Hermle C600U machine tool.

The workpiece coordinate system,  $\mathbf{P}_W=(x,y,z)$ , and machine coordinate system,  $\mathbf{P}_M=(X,Y,Z)$ , are defined as the intersection of the axes of rotations for the  $A$  and  $C$  rotary axes. The  $(X,Y,Z)$  directions of the machine coordinate frame align with the  $(X,Y,Z)$  translational movements along their corresponding guide-ways. When

the  $A$  and  $C$  rotary axes are set to zero the  $(x,y,z)$  directions in the workpiece coordinate system agree with the machine coordinate system.

In terms of machine coordinates the cutter is always aligned in the  $Z$ -direction. However in workpiece coordinates, the cutter orientation depends upon the angles of the rotary axes. A unit vector,  $\mathbf{O}_W$ , is used to describe this orientation. This relationship is given by the kinematic chain connecting the two coordinate frames:

$$\mathbf{O}_M = \mathbf{R}_x(A) \mathbf{R}_z(C) \mathbf{O}_W$$

where  $A$  and  $C$  are the angles of the rotary axes,  $\mathbf{R}_i(j)$  is the rotation matrix along the  $i$  axis of angle  $j$  and  $\mathbf{O}_M = (0,0,1)^T$ .

Certain orientations correspond to a singularity, these occur when  $\mathbf{O}_W = (0,0,1)^T$ . This is due to the fact that when the cutter is oriented at  $(0,0,1)^T$  it is possible to spin the  $C$ -axis and follow a circle in the  $XY$  plane centered on the  $C$ -axis of rotation without affecting the output, i.e. position and orientation of the cutter with respect to the workpiece. This singularity is characterised purely by the orientation of the cutter with respect to the workpiece. Therefore only orientations of the tool are considered here.

The orientation of the cutter is entirely dependent upon the  $A$  and  $C$  axis configuration. To examine the effect that a change in orientation has on changes of the configuration, the orientation is described as a function of time,  $\mathbf{O}_W(t)$  for  $t \in [t_0, t_1]$ . The total orientation change can be found by summing the speed of orientation change over time. This corresponds to the length of the path traced out by  $\mathbf{O}_W(t)$  on the unit sphere.

The derivative vector of the orientation lies on the plane normal to the sphere at  $\mathbf{O}_W$ . This vector can be split into two components,  $\theta_v$  and  $\theta_h$ , describing movements for the  $A$  and  $C$  axes respectively (Figure 2). The  $\theta_v$  component acts in the *vertical* direction,  $\hat{\mathbf{t}}_v$ , towards the singularity,  $\mathbf{O}_M = (0,0,1)^T$ . The  $\theta_h$  acts in the *horizontal* direction,  $\hat{\mathbf{t}}_h$ , orthogonal to  $\hat{\mathbf{t}}_v$ . These unit vectors can be found via the following equation:

$$\hat{\mathbf{t}}_v = \frac{\mathbf{O}_M - (\mathbf{O}_M \cdot \mathbf{O}_W) \mathbf{O}_W}{\|\mathbf{O}_M - (\mathbf{O}_M \cdot \mathbf{O}_W) \mathbf{O}_W\|}$$

$$\hat{\mathbf{t}}_h = \hat{\mathbf{t}}_v \times \mathbf{O}_W,$$

$$\text{and } \dot{\mathbf{O}}_W = \theta_v \hat{\mathbf{t}}_v + \theta_h \hat{\mathbf{t}}_h.$$

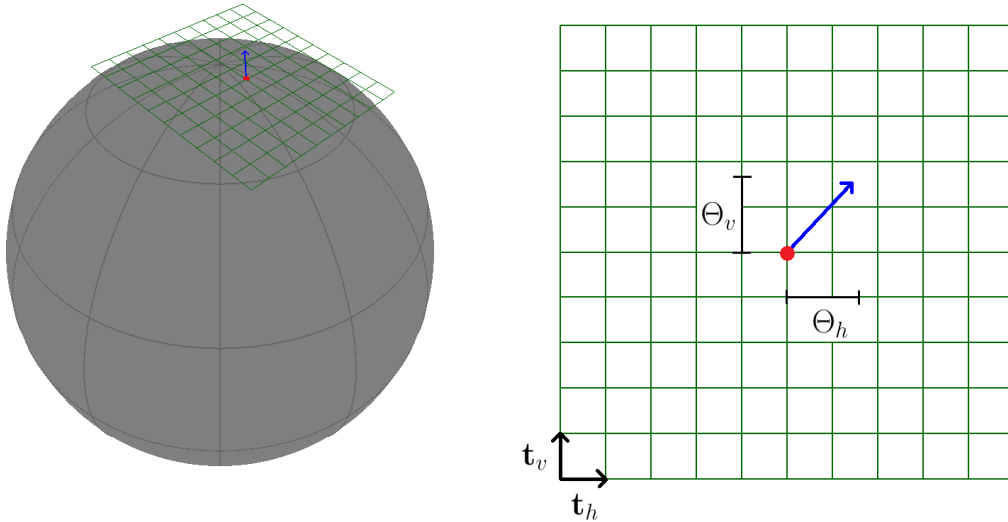


Figure 2. Left: Orientation vector  $\mathbf{O}_W$  (red) and the derivative  $\dot{\mathbf{O}}_W$  (blue). Right: Tangent plane used to form the vertical and horizontal components  $\Theta_v$  and  $\Theta_h$ .

Consider a tool path with purely vertical orientation changes. This corresponds to movements in just the  $A$ -axis. The total change in orientation angle is the length of the path traced out by  $\mathbf{O}_W$  on the unit sphere which forms an arc segment of the unit circle. Now consider a tool path with purely

vertical orientation changes. This corresponds to movements in just the  $C$ -axis. The total change in orientation angle are now arc segments of horizontal circles.

A change in the  $A$ -axis thus corresponds to the length of the arc segment of the unit circle. Changes in the  $C$ -axis however are measured relative to the size (circumference) of the horizontal circles. The size of these circles relative to the unit circle is  $\sin(A)$ . Therefore:

$$\frac{\partial A}{\partial \theta_v} = 1, \quad \frac{\partial C}{\partial \theta_v} = \frac{1}{\sin(A)}. \quad (1)$$

From equation (1) it can be seen that the amount of movement in the  $A, C$ -axes does not depend solely upon the change of orientation,  $\theta$ , but rather the horizontal and vertical components. Furthermore the amount of  $C$ -axis movement depends upon the  $A$ -axis position. The amount of movement required by the  $C$ -axis for a given horizontal component becomes infinite as  $\sin(A) \rightarrow 0$ . This explains the undesired machine behaviour at the singularity. Furthermore, since the effect is quantified by the  $\frac{1}{\sin(A)}$  term, actions can be taken to minimise this.

### 3 REORIENTATION STRATEGY

This section proposes an approach to singularity avoidance by reorientation of the workpiece. Rather than locally reorienting the cutter with respect to the workpiece it is possible to reorient the whole workpiece with respect to the machine. This can be achieved through the use of a jig. The CAM tool path needs to be correspondingly reoriented but there is no need to regenerate the paths. The reorientation procedure is now outlined.

The workpiece is mounted on a jig (Figure 3) which allows it to tilt about a horizontal axis which is taken as a local  $x$ -axis. The rotation angle is  $\theta_x$ . In addition the workpiece can rotate about the axis perpendicular to the plane of the jig. This is the local  $z$ -axis. The rotation angle is  $\theta_z$ .

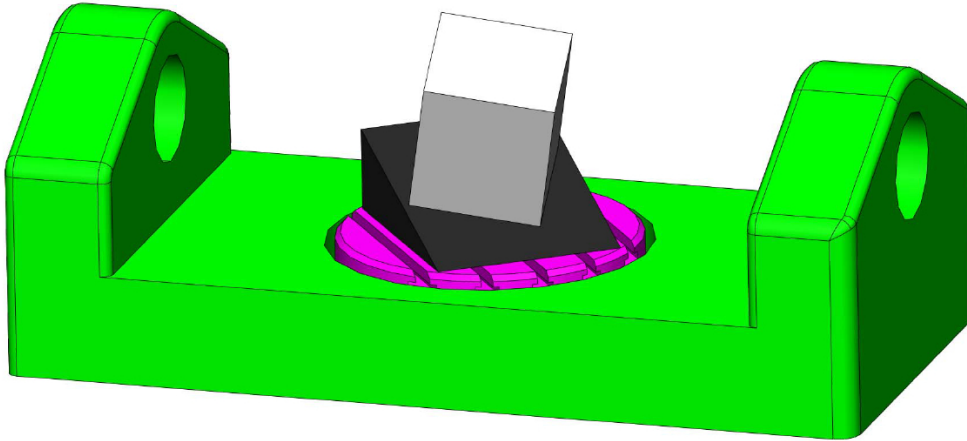


Figure 3. Reorienting the workpiece (white) via the use of a jig (black).

The following kinematic chain describes the link between orientations of the cutter with respect to the workpiece without the jig,  $\mathbf{O}_w$ , and with the jig,  $\overline{\mathbf{O}_w}$ . It can then be used to reorient the original CAM tool path:

$$\overline{\mathbf{O}_w} = \mathbf{R}_x(\theta_x) \mathbf{R}_z(\theta_z) \mathbf{O}_w.$$

Singularities of the modified tool path now occur when the reoriented cutter is perpendicular to the machine bed. The orientation of the singularity in the original tool path thus corresponds to the vector:

$$\mathbf{s} = \mathbf{R}_z(-\theta_z) \mathbf{R}_x(-\theta_x) (0, 0, 1)^T.$$

This reorientation procedure has had the effect of changing the singularity for the original tool path. Furthermore, by altering the angles  $\theta_x$  and  $\theta_z$  the singularity can be oriented with respect to the workpiece in an arbitrary direction.

Existing singularity avoidance strategies are deemed acceptable if the tool path exceeds a tolerable distance away from the singularity. Examining the behaviour of  $C$ -axis movement local to the singularity, equation (1), suggests that this distance should be measured with respect to the angle the tool path forms with the singular configuration. This can be addressed with the reorientation procedure. Furthermore, the choice in parameters  $\theta_x$  and  $\theta_z$  provides scope for finding optimal values.

#### 4 EXAMPLE

Consider the following example of a tool path which passes close to the singularity (Figure 4). The cutter location,  $\mathbf{P}_w$ , and orientation,  $\mathbf{O}_w$ , are given as a function of time for  $t \in [0,6]$ . Position is measured in mm and time in seconds. The example corresponds to a 50mm cut with a cutting feed rate of 500mm/min. The orientation of the cutter follows a semi-circle on the unit sphere by rotating uniformly about the circle centre  $\mathbf{v}$ .

$$\mathbf{P}_w(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \frac{25}{6}(1-t) \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} + \frac{25}{6}t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},$$

$$\mathbf{O}_w(t) = \begin{pmatrix} l(t) \\ j(t) \\ k(t) \end{pmatrix} = \mathbf{R}_v \left( \frac{\pi t}{6} \right) \cdot \frac{1}{33} \begin{pmatrix} 4 \\ -7 \\ 32 \end{pmatrix}, \quad \text{where } \mathbf{v} = \frac{1}{31} \begin{pmatrix} -5 \\ -6 \\ 30 \end{pmatrix}.$$

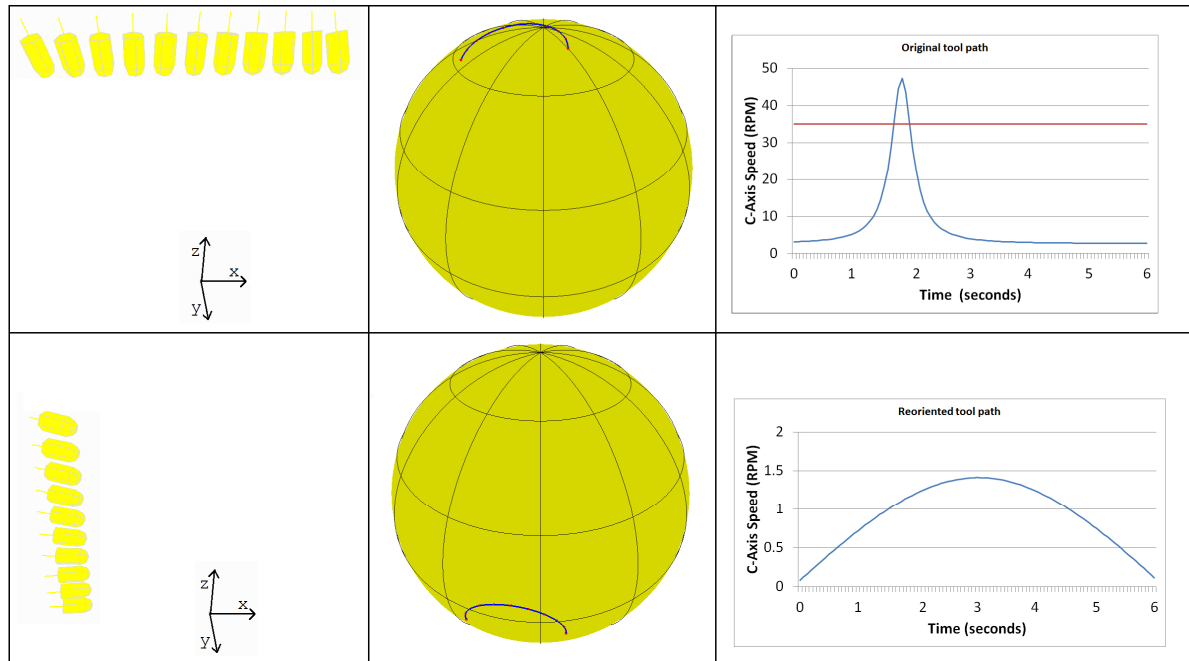


Figure 4. Left: the original (top) and reoriented (bottom) tool paths. Centre: the orientation  $\mathbf{O}_w(t)$  visualised on the sphere. Right: speed of the  $C$ -axis movement.

At its zenith the tool path is  $1.6^\circ$  away from the singularity. To maintain cutting feed rate the  $C$ -axis would have to be spinning at 47.4rpm. This is not possible with the Hermle C600U and therefore a slowdown in cutting feed rate is experienced. To overcome this a reorientation of the workpiece can be applied (see Section 2). Sampling the parameters  $\theta_x$  and  $\theta_z$  at  $0.01^\circ$  intervals, the solution furthest

from the singularity is achieved when  $\theta_x=86.6^\circ$  and  $\theta_z=5.2^\circ$ . The maximum speed of the C-axis required to maintain the feed is reduced from 47.4rpm to 1.4rpm. The reoriented tool path is given in figure 3.

## 5 CONCLUSIONS

When a tool path configuration gets close to a singularity undesired machine axes behaviour is observed. By examining the machine axes movement for an arbitrary tool path, the singular behaviour was identified as a large increase in the speed. The increase depends upon the angle formed with the singular configuration. To prevent the undesired machine behaviour the singularity should be avoided by increasing this angle. A reorientation strategy that uses a jig to reorient the workpiece on the machine bed is proposed. This has the effect of moving the singularity into a safe position. This can be used to move singularities away from the CAM tool path resulting in a reduction in the speed of machine axes.

The mathematical model of machine axes movement provides a framework which can be used to determine how close the tool path can be to a singularity. Singularity region tolerances based upon machine accuracy tolerances appear to have no link to the underlying causality. For example a tolerance of  $0.00278^\circ$  is given in (Affouard *et al.* 2004). This corresponds to an increase in the speed of machine axes movement by a factor of 20,000. A new approach to singularity region definition should be considered that looks at the effect on speed/acceleration/jerk. This would be useful in ascertaining suitable tolerances for individual machine tools.

## ACKNOWLEDGMENTS

The research is supported by the EPSRC research council (EP/L010321/1). The authors would also like to thank Delcam PLC for supporting the research presented in this paper.

## REFERENCES

- Affouard, A., E. Duc, C.Lartigue, J.-M.Langeron, and P. Bourdet. 2004. *Avoiding 5-axis singularities using tool path deformation*, International Journal of Machine Tools and Manufacture 44, 415-425.
- Hermle. 1999. *Hermle C600 series brochure*, Hermle, Gosheim, Germany.
- Lavernhe, S., Y.Quinsat, and C. Lartigue. 2010. *Model for the prediction of 3D surface topography in 5-axis milling*, International Journal of Advanced Manufacturing Technology 51, 915-924.
- Lin, Z., J. Fu, H. Shen, and W. Gan. 2014. *Non-singular tool path planning by translating tool orientations in C-space*, International Journal of Advanced Manufacturing Technology 71, 1835-1848.
- Sørby, K. 2007. *Inverse kinematics of five-axis machines near singular configurations*, International Journal of Machine Tools and Manufacture 47, 299-306.
- Yang, J., and Y. Altintas. 2013. *Generalized kinematics of five-axis serial machines with non-singular tool path generation*, International journal of machine tools and manufacture 75, 119-132.
- Zlatanov, D., R. G. Fenton, and B. Benhabib. 1994. *Singularity analysis of mechanisms and robots via a velocity-equation model of the instantaneous kinematics*, IEEE International Conference on Robotics and Automation.